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Optimal Pollution Control in a Dynamic Multi-echelon Supply Chain

Xavier Brusset · Aida Jebali · Davide La Torre · Shumail Mazahir

Résumé en français

La tarification du carbone couvrira bientôt un cinquième des émissions mondiales. Les entreprises doivent s'attendre à une réglementation toujours plus intense (1 900 textes législatifs sur le climat dans le monde, dont les deux tiers ont été adoptés au cours des dix dernières années), généralement sous l'impulsion des parties prenantes et des actionnaires activistes. Compte tenu de l'impact environnemental du transport de marchandises, comment un responsable logistique peut-il contribuer à l'effort de réduction de ces émissions : 28% des émissions totales de gaz à effet de serre aux Etats-Unis et 30% dans l'Union européenne, le transport routier représentant respectivement 82% et 72% de ces totaux?

Dans cette recherche, nous montrons comment un planificateur de transport logistique optimise les déplacements à effectuer tout en minimisant les émissions de gaz à effet de serre dans un cadre dynamique. Aujourd'hui, les chaînes d'approvisionnement doivent internaliser l'impact du transport sur l'environnement dans leurs modèles de coûts. Les gestionnaires doivent tenir compte de l'ensemble des émissions de gaz à effet de serre, du coût du transport et de la satisfaction de la demande.

Habituellement, un réseau de chaîne d'approvisionnement implique des centaines de fournisseurs, des dizaines d'entrepôts ou d'usines et des centaines de points de livraison. Dans cette étude, nous modélisons le double objectif de minimiser la pollution tout en minimisant le coût de transport sous la contrainte de la satisfaction de la demande dans le temps pour un nombre quelconque de fournisseurs et de points de livraison et un entrepôt central. En utilisant un modèle de théorie du contrôle optimal, nous résolvons ce modèle directement lorsque les exigences de transport des fournisseurs et la demande des points de vente sont soit déterministes (prévisions de transport à horizon glissant), soit décrites par un processus stochastique quelconque dans lequel la demande attendue peut ou non varier dans le temps.

La solution tient compte des coûts de pénalisation pour les livraisons manquées, des coûts des stocks et des émissions de gaz à effet de serre qui peuvent varier dans le temps et l'espace. Nous montrons comment un gestionnaire peut atténuer les émissions avec des paramètres convenablement définis et ainsi choisir les bonnes livraisons, aux bons moments, réduire les émissions de gaz à effet de serre et réussir à atteindre le niveau de service de transport attendu tout en réduisant les coûts de maintien des stocks. Une illustration numérique présente quelques scénarios.

L'utilisation de la théorie du contrôle optimal permet aux modèles d'accepter n'importe quelle évolution continue (y compris les mouvements browniens géométriques) de la production des fournisseurs, de la demande des clients, des pénalités pour ne pas être en mesure d'effectuer le transport, etc. Une telle flexibilité dans l'utilisation des entrées et des variables dans les problèmes de transport n'a jamais été présentée auparavant dans la recherche scientifique centrée sur le transport, à l'exception peut-être des travaux de Tapiero dans les années 1970 (Tapiero 1971, 1972,

Tapiero et Soliman 1972). Nous prétendons qu'elle sera d'une aide considérable pour les gestionnaires ou du moins pour les éditeurs de logiciels logistiques. Contrairement à d'autres méthodes de programmation linéaire ou quadratique utilisées dans d'autres cas d'optimisation du transport, la solution fournie par la Théorie du Contrôle Optimal est la solution optimale et non une approximation, obtenue en une seule étape de calcul.

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Optimal Pollution Control in a Dynamic Multi-echelon Supply Chain

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Abstract

Today supply chains must internalize the impact of transport on the environment in their cost models. Therefore, managers must reduce green house gas emissions while striving to increase cost efficiency and satisfy demand. Our multi-echelon supply chain model minimizes pollution and cost while trying to achieve the best match between supply and demand over time. Three supply chain network configurations are investigated. Two of them are two-echelon: the first involves several suppliers and one warehouse, while the second involves one warehouse and several retail stores. The third network configuration is a three-echelon supply chain including multiple suppliers, one distribution center, and several retail stores. Using optimal control theory, we derive closed form solutions in such multi-echelon supply chain planning problems with consideration of pollution. This approach extends in a new direction the literature in operations and transport management by simultaneously addressing demand, supply as well as the greenhouse gas emissions that continuously vary in time and location. The proposed model provides a decision maker with the optimal choice of right deliveries, right times, while minimizing green house gas emissions. A numerical illustration presents some insights.

Keywords Pollution regulation · Optimal control theory · Transport optimization · Supply chain

1 Introduction

Carbon pricing will soon cover a fifth of the world's emissions. Companies should expect ever more intense regulation (1,900 pieces of climate legislation around the world, two-thirds were enacted in the past ten years [1]), generally driven by stakeholders and activist shareholders [2]. Given the environmental impact of freight transportation, how can a logistics manager contribute to the effort in reducing such emissions 28% of total greenhouse gas emissions in the USA and 30% in the European Union, with road transport representing 82% and 72% of those totals, respectively [3, 4]?

To help logistics managers, as central planning decision makers, we present three multi-echelon supply chain models that jointly optimize the delivered quantities in order to satisfy as much as possible the demand while minimizing CO2 emissions. Two models consider two-echelon supply chains: the first involves several suppliers and one warehouse, while the second involves one warehouse and several retail stores. The third, thereafter referred to as an integrated supply chain, is three-echelon and includes multiple suppliers, one distribution center, and several retail stores. Noticeably, these three types of supply chain network configurations cover most of the cases that a logistics manager has to deal with. We also provide the solutions to two particular configurations: (a) a network of suppliers to a warehouse when lead times must be taken into account; (b) a network of retail stores served by a distribution center when the former have an inventory policy in place.

Unlike the conventional models that optimize (often in two steps) either pollution emission alone, or in location-dependent or time-dependent settings, our model, proposed for the integrated supply chain, for example, offers a direct

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and optimal distribution plan that determines the quantity to transport from all suppliers and deliver to all retail stores connected to a distribution center while trying to reduce pollution that continuously vary over time.

Our research fills a gap in the transport and supply chain management literature by offering a model that captures simultaneously the features of transport scheduling, pollution control, and demand satisfaction.

After reviewing the literature to which this research contributes in the next section, we present our contribution and the model in Sect. 3. The models capture three network configurations: several suppliers and one warehouse in subsection 3.1, one warehouse and several retail stores in subsection 3.2, a whole network centered on a single distribution center in subsection 3.3. In Sect. 5, we present a numerical illustration of the model. Section 4 introduces two extensions: one with lead times, another with inventories at retail stores. Managerial insights and conclusions are spelt out in Sect. 6.

2 Literature Review

Our work lies at the interface of supply chain management and sustainable operations and is related to several streams of literature. The first stream of work is related to co-ordinated logistics and transportation [5–7]. In this stream of literature, joint inventory and transportation decisions are taken in a single- or two-echelon supply chain. We extend this by (i) presenting a model in continuous time, (ii) offering a three-echelon supply chain with multiple suppliers and retailers (iii) incorporating pollution control, (iv) taking into account varying pollution in time and location. We consider uncapacitated transportation to accommodate the possibility of shared transportation initiatives.

The second stream of relevant work stems from sustainable transportation that has been devoted to the study of transportation decisions from an environmental perspective [8, 9]. Jabali et al. [8] introduce carbon emissions in the context of a time-dependent vehicle routing problem. We restrict to supply chain decisions within the context of time-dependent emissions as studied in [10]. This context is essential from the perspective of air pollution in urban centers that interfere with air quality. Many cities reduce speed limits on their highways and impose restrictions on vehicle entry due to air quality. To incorporate time-dependent effects, we model the problem in continuous time using an optimal control theory framework.

A third stream of literature combines the objective of reducing emissions with network decisions [11–13]. The review in [14] illustrates the scarcity of models that study the implications of carbon footprint policies on multi-echelon supply chains. A key distinguishing feature of our work is the presentation of a model which, on the one hand, enables carbon footprint optimization and, on the other hand, provides closed form solutions. Similarly to [15], we model demand from stores and supply from suppliers as being continuous in time. This is more appropriate given the growth of e-commerce and fast delivery, on the one hand, and the frequent delivery schedules to minimize inventory at warehouses coupled with unexpected arrival and transport times on the other. Demand pricing [16], transportation, revenue management [17–19], spare part supply chains [20], operational systems [21], fleet repositioning problems [22] have a long tradition of being solved by modeling time as continuous [23].

The fourth stream of literature stems from the popularity of optimal control theory in operations management literature as a mathematical optimization tool because of the ability to address, usually in closed form, optimization problems combining several objectives in time [24]. It has been applied for scheduling and planning problems to obtain tractable solutions [25], in modeling humanitarian operations [26], service operations [27], production inventory systems [28–31], or pollution control [32]. Papier [30] uses optimal control methodology to manage production-inventory decisions in the presence of peak-load electricity pricing policies. They optimize joint inventory and back-ordering decisions amid accumulating electricity costs. We are optimizing the product transport allocation decisions with accumulating pollution levels in time.

Since the seminal work of Charles Tapiero [33–35], we found no research advancement on dynamic transportation models using optimal control theory. In a way, we are rekindling the interest in the work of Charles Tapiero given its stark relevance for optimization under pollution and service level constraints.

3 Contribution and Model

Our contribution uses optimal control theory to show how best to control CO2 emissions while trying to match the demand with the supply. There are several advantages of using this theory.

- It captures a more accurate representation of existing supply chains with planned demand and supply schedules than the traditional or two-stage decision-making approaches.
- It generates closed-form solutions so as to enable comparative statics with reference to the variables of interest, making it possible to identify critical supply chain variables in a pollution control exercise.

- It enables the inclusion of time-dependent pollution emission, which also provides a context for urban pollution as pollution peaks at certain hours and seasons [10].
- It can accommodate any time-continuous function to describe the demand, the supply, the transport cost, as well as the environmental impact of transport per location.
- Both transport cost and CO2 emissions are optimized in time and space.
- Our models can solve a very large network in number of suppliers and retail stores with an insignificant computational effort, favoring frequent adjustments.
- Models using this theory can take into account any open-loop consideration since they can be re-evaluated to take feedback into account.

In all supply chain models, the closed-form solutions allow to find the optimal transport schedule in terms of pollution and level of demand satisfaction (also referred to as service level) given a set of demands over a time horizon. The models are generic and are adaptable to almost any form of pollution from road transport. As noted above, we consider three supply chain network configurations (i) multiple suppliers and a single warehouse, (ii) single warehouse, multiple retail stores, and (iii) integrated supply chain network with multiple suppliers, a distribution center and multiple retail stores expressing demand which has to be fulfilled.

Given that the obtained closed-form solution is not computationally intensive, the supply chain planner can benefit from it to optimize jointly transportation schedule and CO2 emissions. The open-loop problem is dealt with as in [36] by re-evaluating the model as feedback from demand is taken into account in the schedule.

We have organized our study as follows. For each of the considered supply chains, we provide a closed-form solution. We present in subsection 3.1 a network with multiple suppliers and a single warehouse as destination. In subsection 3.2, we present the optimal delivery schedule from a single warehouse to various retail stores. Finally, subsection 3.3 offers a three-echelon supply chain model with multiple suppliers, one distribution center, and a cluster of retail stores.

3.1 Multiple Suppliers, Single Warehouse Optimal Pick-up Schedule

We consider a dynamic transportation model that aims at finding the optimal transportation schedule to load goods from suppliers and deliver them to a single warehouse (or a production facility) by minimizing the cost of transportation, the penalty for not matching the supply with the demand, and the amount of CO2 emissions over a planning horizon, $[0, T]$. Obviously, the demand at the level of the warehouse represents the aggregated requirements from the downstream echelons in the supply chain. We assume that a contract quantity is committed between the warehouse and each supplier. According to this agreement, any shipped quantity from the supplier to the warehouse below or beyond the contract quantity is penalized. The contract quantity is assumed known, and its determination is beyond the scope of this paper. At this level, it is worth noting that each supplier, whenever needed, has enough capacity to provide the warehouse with a quantity of product that is greater than the contract quantity.

The objective function includes four terms: the first one is the transportation cost. The second and third ones describe the costs for not matching the shipped quantity with the contract quantity committed with the supplier, and the demand, respectively. The fourth term tries to minimize to total level of pollution at the end of the planning horizon. CO2 emissions accumulate through a differential equation that accounts for the amount of shipped goods from each supplier given its location and natural cleaning rate (for more general pollution equations one can see [37–39]).

We summarize the notation used for all models in Table 1.

If we neglect the travel time with respect to the considered time unit, the objective function to be minimized reads as:

$$\min_{x_i(t),P(t)} \sum_{i=1}^{N} \int_0^T c_i(t)x_i(t)dt + \frac{1}{2}\sum_{i=1}^{N}\int_0^T \alpha_i(t)\big(x_i(t)-s_i(t)\big)^2 dt + \frac{1}{2}\int_0^T \beta(t)\left(\sum_{i=1}^{N} x_i(t)-\xi(t)\right)^2 dt + \theta P(T), \tag{1}$$

subject to

$$\begin{aligned} &\dot{P}(t) = \sum_{i=1}^{N} \gamma_i(t)x_i(t) - \delta_P P(t) \\ &P(0) \geq 0, \\ &x_i(t) \geq 0, \qquad i = 1 \dots N,\ t \in [0, T]. \end{aligned} \tag{2}$$

Theorem 1 *Let us define*:

$$\Omega(t) := \begin{pmatrix} \alpha_1(t)+\beta(t) & \beta(t) & \dots & \beta(t) \\ \beta(t) & \alpha_2(t)+\beta(t) & \dots & \beta(t) \\ \dots & \dots & \dots & \dots \\ \beta(t) & \beta(t) & \dots & \alpha_N(t)+\beta(t) \end{pmatrix},$$

$$X(t) := \begin{pmatrix} x_1(t) \\ x_2(t) \\ \dots \\ x_N(t) \end{pmatrix}, \qquad C(t) := \begin{pmatrix} c_1(t) \\ c_2(t) \\ \dots \\ c_N(t) \end{pmatrix},$$

Table 1 Notations of all models

N	number of suppliers,
M	number of retail stores,
T	length of the planning horizon,
$c_i(t)$	unit transportation cost from supplier i to the warehouse/distribution center at time t,
$\bar{c}_j(t)$	unit transportation cost from the warehouse/distribution center to retail store j at time t,
$x_i(t)$	quantity shipped from supplier i to the warehouse/distribution center at time t,
$s_i(t)$	contract quantity committed with supplier i at time t,
$\xi_j(t)$	demand at the retail store j at time t,
$\xi(t)$	demand at the warehouse/distribution center at time t,
$\bar{\xi}(t)$	quantity of product available at the warehouse at time t,
$\alpha_i(t)$	penalty for not matching the shipped quantity with the contract quantity committed with supplier i at time t,
$\bar{\alpha}_j(t)$	penalty for not matching the delivered quantity to retail store j with the demand at time t,
$\beta(t)$	penalty for not matching the total shipped quantity from the suppliers with the demand of the warehouse/distribution center at time t,
$\bar{\beta}(t)$	penalty for not matching the total shipped quantity from the suppliers to the total delivered quantity to the retail stores at time t,
$\theta, \bar{\theta}$	trade-off parameter which states the importance of the pollution level at $t = T$,
$P(t)$	total air pollution emission in time t,
$\gamma_i(t)$	air pollution emission per unit of product transported between location i and the warehouse/distribution center at time t,
$\bar{\gamma}_j(t)$	air pollution emission per unit of product transported between the warehouse/distribution center and the retail store j at time t,
$\delta_P, \bar{\delta_P}$	depreciation rate of CO2 emissions, that is the natural cleaning rate of the level of pollution,
$y_j(t)$	quantity of product to be delivered from the warehouse/distribution center to the retail store j,
L	lead time,
$\eta_j(t)$	amount of inventory at the retail store j that is subtracted to the demand at time t. The expression $\int_0^T \eta_j(t)dt = \bar{\eta}_j$ describes the total inventory available at the j^{th} retail store at the beginning of the considered planning horizon (i.e., $t = 0$). This inventory results from the decisions made while running the model for the previous planning horizon.

$$S(t) := \begin{pmatrix} s_1(t)\alpha_1(t) + \xi(t)\beta(t) \\ s_2(t)\alpha_2(t) + \xi(t)\beta(t) \\ \dots \\ s_N(t)\alpha_N(t) + \xi(t)\beta(t) \end{pmatrix}, \qquad \Gamma(t) := \begin{pmatrix} \gamma_1(t) \\ \gamma_2(t) \\ \dots \\ \gamma_N(t) \end{pmatrix}.$$

Suppose that (component-wise) $S(t) \geq C(t) + \theta e^{\delta_P(t-T)}\Gamma(t)$, $\Gamma(t) \geq 0$, *and* $\Omega^{-1}(t) \geq 0$ *for any* $t \in [0, T]$.

Then, the optimal solution to the optimal control model is the pair $(X(t), P(t))$ *given by*:

$$X(t) = \Omega(t)^{-1}\left[S(t) - C(t) - \theta e^{\delta_P(t-T)}\Gamma(t)\right], \tag{3}$$

$$P(t) = e^{-\delta_P t}\left[\int_0^t \Gamma(s)^T X(s) e^{\delta_P s} ds + P_0\right], \tag{4}$$

and are therefore always positive.

The proof of this and other theorems are relegated to .

Remark 1 The expression of P can be seen as a first-order Taylor linear approximation of any nonlinear model around the pollution-free equilibrium. Therefore, its validity is general as long as it is used for local dynamic analysis. More general optimal control models involving multi-objective criteria, spatial dimension, stochastic shock, uncertainty, abatement policy are presented in [40–43].

Remark 2 Suppose that the equation of P is subject to an exogenous factor $W(t)$, driven by a classical Wiener process as in the following expression:

$$dP(t) = \left(\sum_{i=1}^{N} \gamma_i(t)x_i(t) - \delta_P P(t)\right)dt + \sigma P(t)dW(t), \quad P(0) = P_0 \tag{5}$$

where P_0 is a deterministic value, all time-dependent parameters are deterministic, and σ is the stochastic process volatility. In this case, thanks to the linearity of the equation with respect to $P(t)$, we proceed by taking the expected value of both sides, and calculate the expected value of the optimal pollution path as follows:

$$\frac{d\mathbb{E}((P(t))}{dt} = \sum_{i=1}^{N} \gamma_i(t)x_i(t) - \delta_P \mathbb{E}((P(t)), \tag{6}$$

the solution of which is then given by

$$\mathbb{E}((P(t)) = e^{-\delta_P t}\left[\int_0^t \Gamma(s)^T X(s) e^{\delta_P s} ds + P_0\right]. \tag{7}$$

To compute the optimal solution, the supply chain planner must input the planned demand which triggers the delivery/transport requirements from the suppliers. The delivered quantity from one supplier should be matched as much as possible with the committed contract quantity. Any mismatch between the delivered quantity and the demand, and the delivered quantity by one supplier and the contracted quantity is penalized. As time goes by, she uses the rolling horizon forecast to recalculate the optimal solution.

3.2 Single Warehouse, Multiple Retail Stores Optimal Delivery Schedule

In this section, we determine the optimal delivery schedule of goods from one warehouse to M different retail stores (or delivery points). The proposed model aims at minimizing an objective function composed by four different terms, namely the cost of delivery (transportation cost), the penalty for not matching the demand with the supply in the retail stores or at the warehouse, and the total level of pollution at T. As in the previous case, if we neglect the travel time with respect to the considered time unit, the model reads as:

$$\min_{y_j(t),P(t)} \sum_{j=1}^M \int_0^T \bar{c}_j(t) y_j(t) dt + \frac{1}{2}\sum_{j=1}^M \int_0^T \bar{\alpha}_j(t)\big(y_j(t) - \xi_j(t)\big)^2 dt$$
$$+ \frac{1}{2}\int_0^T \bar{\beta}(t)\left(\sum_{j=1}^M y_j(t) - \bar{\xi}(t)\right)^2 dt + \bar{\theta}P(T), \tag{8}$$

subject to

$$\begin{aligned} &\dot{P}(t) = \sum_{j=1}^M \bar{\gamma}_j(t) y_j(t) - \bar{\delta}_P P(t),\\ &P(0) \geq 0,\\ &y_j(t) \geq 0, \quad j = 1 \ldots M, t \in [0,T]. \end{aligned} \tag{9}$$

Similarly to the previous case, one can prove that the optimal solution is provided by the following expressions.

Theorem 2 *Let us define*:

$$\bar{\Omega}(t) := \begin{pmatrix} \bar{\alpha}_1(t) + \bar{\beta}(t) & \bar{\beta}(t) & \ldots & \bar{\beta}(t)\\ \bar{\beta}(t) & \bar{\alpha}_2(t) + \bar{\beta}(t) & \ldots & \bar{\beta}(t)\\ \ldots & \ldots & \ldots & \ldots\\ \bar{\beta}(t) & \bar{\beta}(t) & \ldots & \bar{\alpha}_M(t) + \bar{\beta}(t) \end{pmatrix},$$

$$Y(t) := \begin{pmatrix} y_1(t)\\ y_2(t)\\ \ldots\\ y_M(t) \end{pmatrix}, \qquad \bar{C}(t) := \begin{pmatrix} \bar{c}_1(t)\\ \bar{c}_2(t)\\ \ldots\\ \bar{c}_M(t) \end{pmatrix},$$

$$\bar{S}(t) := \begin{pmatrix} \xi_1(t)\bar{\alpha}_1(t) + \bar{\beta}(t)\bar{\xi}(t)\\ \xi_2(t)\bar{\alpha}_2(t) + \bar{\beta}(t)\bar{\xi}(t)\\ \ldots\\ \xi_M(t)\bar{\alpha}_M(t) + \bar{\beta}(t)\bar{\xi}(t) \end{pmatrix}, \text{ and } \bar{\Gamma}(t) := \begin{pmatrix} \bar{\gamma}_1(t)\\ \bar{\gamma}_2(t)\\ \ldots\\ \bar{\gamma}_M(t) \end{pmatrix}.$$

Suppose that (component-wise) $\bar{S}(t) \geq \bar{C}(t) + \bar{\theta}e^{\bar{\delta}_P(t-T)}\bar{\Gamma}(t)$, $\bar{\Gamma}(t) \geq 0$, *and* $\bar{\Omega}^{-1}(t) \geq 0$ *for any* $t \in [0,T]$.

Then, the optimal solution to the optimal control model is the pair $(Y(t), P(t))$ *given by*:

$$Y(t) = \bar{\Omega}(t)^{-1}\left[\bar{S}(t) - \bar{C}(t) - \bar{\theta}e^{\bar{\delta}_P(t-T)}\bar{\Gamma}(t)\right], \tag{10}$$

$$P(t) = e^{-\bar{\delta}_P t}\left[\int_0^t \bar{\Gamma}(s)^T Y(s) e^{\delta_P s} ds + P_0\right], \tag{11}$$

which are always positive.

The supply chain planner can determine the optimal delivery schedule by plugging in the model the planned demand from retail stores. Given that the model accepts varying pollution in time and location, she can measure and control the corresponding pollution and transport service level according to the penalties that she keys in as exogenous parameters.

3.3 Integrated Model

Having presented two models: one where multiple suppliers deliver to a single warehouse and one where one warehouse delivers to multiple retail stores (or delivery points), we now consider a three-stage supply chain composed of several suppliers, a distribution center, and a cluster of retail stores. The supply chain planner should decide on the quantity of product to pick up from a pre-selected panel of suppliers in order to satisfy as much as possible the demand at different retail stores. The products are shipped from the suppliers and cross-docked at a distribution center before being delivered to retail stores. The optimization function is then:

$$\begin{aligned} \min_{x_i(t),y_j(t),P(t)} & \sum_{i=1}^{N} \int_0^T c_i(t)x_i(t)dt + \sum_{j=1}^{M} \int_0^T \bar{c}_j(t)y_j(t)dt \\ & + \frac{1}{2}\sum_{i=1}^{N} \int_0^T \alpha_i(t)\big(x_i(t)-s_i(t)\big)^2 dt \\ & + \frac{1}{2}\sum_{j=1}^{M} \int_0^T \bar{\alpha}_j(t)\big(y_j(t)-\xi_j(t)\big)^2 dt \\ & + \frac{1}{2}\int_0^T \beta(t)\left(\sum_{i=1}^{N} x_i(t)-\xi(t)\right)^2 dt \\ & + \frac{1}{2}\int_0^T \bar{\beta}(t)\left(\sum_{j=1}^{M} y_j(t)-\sum_{i=1}^{N} x_i(t)\right)^2 dt \\ & + \theta P(T). \end{aligned} \tag{12}$$

subject to

$$\begin{aligned} & \dot{P}(t) = \sum_{i=1}^{N} \gamma_i(t)x_i(t) + \sum_{j=1}^{M} \bar{\gamma}_j(t)y_j(t) - \delta_P P(t), \\ & P(0) \geq 0, \\ & x_i(t) \geq 0, \quad i = 1 \dots N, t \in [0,T], \\ & y_j(t) \geq 0, \quad j = 1 \dots M, t \in [0,T]. \end{aligned} \tag{13}$$

Theorem 3 *Let us define*:

$$\Omega 1(t) := \begin{pmatrix} A(t) & B(t) \\ C(t) & D(t) \end{pmatrix} \tag{14}$$

with

$$A(t) := \begin{pmatrix} \alpha_1(t)+\beta(t)+\bar{\beta}(t) & \beta(t)+\bar{\beta}(t) & \dots & \beta(t)+\bar{\beta}(t) \\ \beta(t)+\bar{\beta}(t) & \alpha_2(t)+\beta(t)+\bar{\beta}(t) & \dots & \beta(t)+\bar{\beta}(t) \\ \dots & \dots & \dots & \dots \\ \beta(t)+\bar{\beta}(t) & \beta(t)+\bar{\beta}(t) & \dots & \alpha_N(t)+\beta(t)+\bar{\beta}(t) \end{pmatrix}, \tag{15}$$

$$B(t) := \begin{pmatrix} -\bar{\beta}(t) & \dots & -\bar{\beta}(t) \\ -\bar{\beta}(t) & \dots & -\bar{\beta}(t) \\ \dots & \dots & \dots \\ -\bar{\beta}(t) & \dots & -\bar{\beta}(t) \end{pmatrix}, \tag{16}$$

$$C(t) := \begin{pmatrix} -\bar{\beta}(t) & -\bar{\beta}(t) & \dots & -\bar{\beta}(t) \\ \dots & \dots & \dots & \dots \\ -\bar{\beta}(t) & -\bar{\beta}(t) & \dots & -\bar{\beta}(t) \end{pmatrix}, \tag{17}$$

$$D(t) := \begin{pmatrix} \bar{\alpha}_1(t)+\bar{\beta}(t) & \dots & \bar{\beta}(t) \\ \bar{\beta}(t) & \dots & \bar{\beta}(t) \\ \bar{\beta}(t) & \dots & \bar{\alpha}_M(t)+\bar{\beta}(t) \end{pmatrix}, \tag{18}$$

$$U(t) := \begin{pmatrix} x_1(t) \\ x_2(t) \\ \dots \\ x_N(t) \\ y_1(t) \\ y_2(t) \\ \dots \\ y_M(t) \end{pmatrix}, \qquad C1(t) := \begin{pmatrix} c_1(t) \\ c_2(t) \\ \dots \\ c_N(t) \\ \bar{c_1}(t) \\ \bar{c_2}(t) \\ \dots \\ \bar{c_M}(t) \end{pmatrix}, \tag{19}$$

$$S1(t) := \begin{pmatrix} s_1(t)\alpha_1(t)+\xi(t)\beta(t) \\ s_2(t)\alpha_2(t)+\xi(t)\beta(t) \\ \dots \\ s_N(t)\alpha_N(t)+\xi(t)\beta(t) \\ \bar{\alpha}_1(t)\xi_1(t) \\ \bar{\alpha}_2(t)\xi_2(t) \\ \dots \\ \bar{\alpha}_M(t)\xi_M(t) \end{pmatrix}, \text{and } \Gamma 1(t) := \begin{pmatrix} \gamma_1(t) \\ \gamma_2(t) \\ \dots \\ \gamma_N(t) \\ \bar{\gamma}_1(t) \\ \bar{\gamma}_2(t) \\ \dots \\ \bar{\gamma}_M(t) \end{pmatrix}. \tag{20}$$

Suppose that (component-wise) $S1(t) \geq C1(t) + \theta e^{\delta_P(t-T)} \Gamma 1(t), \Gamma 1(t) \geq 0$, *and* $\Omega 1^{-1}(t) \geq 0$ *for any* $t \in [0,T]$.

Then, the optimal solution to the optimal control model is the pair $(U(t), P(t))$ *given by*:

$$U(t) = \Omega 1(t)^{-1}\left[S1(t) - C1(t) - \theta e^{\delta_P(t-T)}\Gamma 1(t)\right], \tag{21}$$

$$P(t) = e^{-\delta_P t}\left[\int_0^t \Gamma 1(s)^T U(s) e^{\delta_P s} ds + P_0\right], \tag{22}$$

which are always positive.

4 Model Extensions

The next two subsections represent interesting variations on the first and the third proposed models, respectively. The first extension considers a delivery lead time in the multiple supplier, single warehouse pick-up scheduling, while the second one accounts for inventory at retail stores.

4.1 Multiple Suppliers, Single Warehouse Pick-up Scheduling with Lead Time

In this model formulation, we include a constant delivery lead time L in the first model. We suppose that the exogenous parameters $c_i(t)$, $\alpha_i(t)$ and $\beta(t)$ are constant over time. The model reads as:

$$\min_{x_i(t),P(t)} \sum_{i=1}^{N} \int_0^{T-L} c_i x_i(t)dt + \frac{1}{2}\sum_{i=1}^{N} \int_0^{T-L} \alpha_i\big(x_i(t)-s_i(t)\big)^2 dt + \frac{1}{2}\int_0^{T-L} \beta\left(\sum_{i=1}^{N} x_i(t) - \xi(t+L)\right)^2 dt + \theta P(T-L), \tag{23}$$

subject to

$$\begin{aligned} &\dot{P}(t) = \sum_{i=1}^{N} \gamma_i(t)x_i(t) - \delta_P P(t), \\ &P(0) \geq 0, \\ &x_i(t) \geq 0, \qquad i = 1 \dots N,\ t \in [0, T-L]. \end{aligned} \tag{24}$$

The following result characterizes the optimal solution of the above model with lead time.

Theorem 4 *Let* Ω, $X(t)$, $C(t)$, *and* $\Gamma(t)$ *be as defined in Theorem 1, whereas*

$$S_L(t) := \begin{pmatrix} s_1(t)\alpha_1 + \xi(t+L)\beta \\ s_2(t)\alpha_2 + \xi(t+L)\beta \\ \dots \\ s_N(t)\alpha_N + \xi(t+L)\beta \end{pmatrix}.$$

Suppose that $S_L(t) \geq C + \theta e^{\delta_P(t-T+L)}\Gamma(t)$, $\Gamma(t) \geq 0$, *and* $\Omega \geq 0$. *Then, the optimal solution to the optimal control model where c*, α_i, β, *and delivery lead time L are constant in time is the pair* $(X(t), P(t))$ *given by*:

$$X(t) = \begin{cases} \Omega^{-1}\left[S_L(t) - C - \theta e^{\delta_P(t-T+L)}\Gamma(t)\right], & t \in [0, T-L], \\ 0 & t \in [T-L, T], \end{cases} \tag{25}$$

$$P(t) = \begin{cases} e^{-\delta_P t}\left[\int_0^t \Gamma(s)^T X(s) e^{\delta_P s} ds + P_0\right], & t \in [0, T-L], \\ e^{-\delta_P t}\left[\int_0^{T-L} \Gamma(s)^T X(s) e^{\delta_P s} ds + P_0\right], & t \in [T-L, T]. \end{cases} \tag{26}$$

4.2 An Integrated Model with Inventory

Here, we consider an inventory available in each retail store at the beginning of the considered planning horizon and an inventory policy which is exogenously given. More specifically, this policy will define how this inventory will be deployed over time and hence contribute to achieve a better match between the supply and the demand at each retail store.

The optimization problem reads:

$$\begin{aligned} \min_{x_i(t),y_j(t),P(t)} & \sum_{i=1}^{N} \int_0^{T} c_i(t)x_i(t)dt + \sum_{j=1}^{M} \int_0^{T} \bar{c}_j(t)y_j(t)dt \\ & + \frac{1}{2}\sum_{i=1}^{N} \int_0^{T} \alpha_i(t)\big(x_i(t)-s_i(t)\big)^2 dt \\ & + \frac{1}{2}\sum_{j=1}^{M} \int_0^{T} \bar{\alpha}_j(t)\big(y_j(t)-(\xi_j(t)-\eta_j(t))\big)^2 dt \\ & + \frac{1}{2}\int_0^{T} \beta(t)\left(\sum_{i=1}^{N} x_i(t) - \sum_{j=1}^{M}(\xi_j(t)-\eta_j(t))\right)^2 dt \\ & + \frac{1}{2}\int_0^{T} \bar{\beta}(t)\left(\sum_{j=1}^{M} y_j(t) - \sum_{i=1}^{N} x_i(t)\right)^2 dt \\ & + \theta P(T), \end{aligned} \tag{27}$$

subject to

$$\begin{aligned} &\dot{P}(t) = \sum_{i=1}^{N} \gamma_i(t)x_i(t) + \sum_{j=1}^{M} \bar{\gamma}_j(t)y_j(t) - \delta_P P(t), \\ &P(0) \geq 0, \\ &x_i(t) \geq 0 \quad i = 1 \dots N, t \in [0, T], \\ &y_j(t) \geq 0 \quad j = 1 \dots M, t \in [0, T]. \end{aligned} \tag{28}$$

It is worth noting that the distribution center is a cross-docking platform. This justifies why the model does not include any inventory at the distribution center at the beginning of the planning horizon. Furthermore, as mentioned above, the inventory level at each retail store at the beginning of a given planning horizon can be determined from the decisions taken while running the proposed model for the preceding planning horizon. From this perspective, running the model for k successive planning horizons $[0,T_1],[T_1,T_2]...[T_{k-1},T_k]$, will allow to build a plan over $[0,T_k]$ that accounts for the intertemporal relationships in inventories.

Theorem 5 *Let* $\Omega 1(t)$, $U(t)$, $C1(t)$, $\Gamma 1(t)$ *be defined as in Theorem 3 and*

$$S11(t) := \begin{pmatrix} s_1(t)\alpha_1(t) + \beta(t)\sum_{j=1}^{M}(\xi_j(t)-\eta_j(t)) \\ s_2(t)\alpha_2(t) + \beta(t)\sum_{j=1}^{M}(\xi_j(t)-\eta_j(t)) \\ \dots \\ s_N(t)\alpha_N(t) + \beta(t)\sum_{j=1}^{M}(\xi_j(t)-\eta_j(t)) \\ \bar{\alpha}_1(t)(\xi_1(t)-\eta_1(t)) \\ \bar{\alpha}_2(t)(\xi_2(t)-\eta_2(t)) \\ \dots \\ \bar{\alpha}_M(t)(\xi_M(t)-\eta_M(t)) \end{pmatrix}. \tag{29}$$

Suppose that (component-wise) $S11(t) \geq C1(t) + \theta e^{\delta_P(t-T)}$ $\Gamma 1(t), \Omega 1(t)^{-1} \geq 0$, *and* $\Gamma 1(t) \geq 0$ *for any* $t \in [0,T]$.

Then, the optimal solution to the optimal control model is the pair $(U(t), P(t))$ *given by*:

$$U(t) = \Omega 1(t)^{-1}\left[S11(t) - C1(t) - \theta e^{\delta_P(t-T)}\Gamma 1(t)\right], \tag{30}$$

$$P(t) = e^{-\delta_P t}\left[\int_0^t \Gamma 1(s)^T U(s) e^{\delta_P s} ds + P_0\right], \tag{31}$$

which are always positive.

5 Illustrative Examples

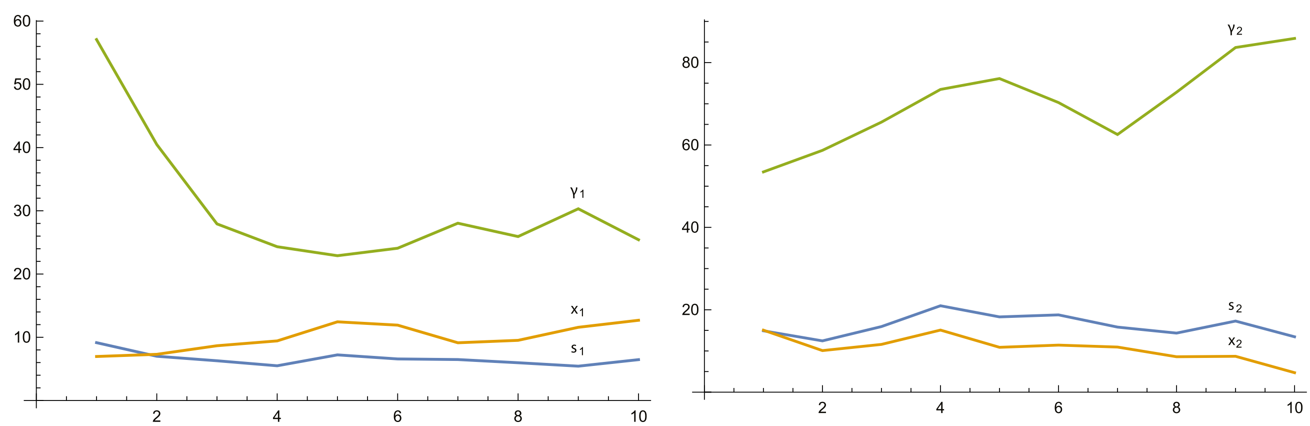
We first represent a numerical illustration of the case where two suppliers have to supply one warehouse. Afterward, we repeat the exercise for the case of a three-stage supply chain including multiple suppliers, a central distribution center and multiple retail stores, using the integrated model.

We use the parameters' values provided in [44].

5.1 One Warehouse, Two Suppliers

Consider the case of a warehouse which has to satisfy demand for one item that two suppliers can provide over a planning horizon [0,10]. Without loss of generality, let $\xi(t) = s_1(t) + s_2(t)$. The CO2 emissions evolution is deterministic and varies in time based on the location of supplier and the shipped quantity. The penalties are fixed over all periods with $\alpha = 5$ and $\beta = 60$. Indeed, in this case, the supply chain planner strives to match the supply with the demand at the warehouse, from either of the suppliers, while the overall impact of CO2 emissions is minimized. Such a configuration may be used to account for the deployed transport mode or route to deliver products from each supplier to the warehouse (hence the difference in CO2 emissions).

As can be observed from Fig. 1, the level of CO2 emissions for the trajectory between the second supplier and the warehouse becomes much more important than from the first supplier toward the end of the planning horizon. Therefore, more cargo is shipped from supplier 1 rather than supplier 2 in order to reduce the overall CO2 emissions.



(a) Graph of CO2 emissions, demand and transport from supplier 1

(b) Graph of CO2 emissions, demand and transport from supplier 2

Fig. 1 Evolution over time of shipped quantities from the suppliers vs. the CO2 emission per unit of product transported from supplier i to the warehouse, $\gamma_i(t)$

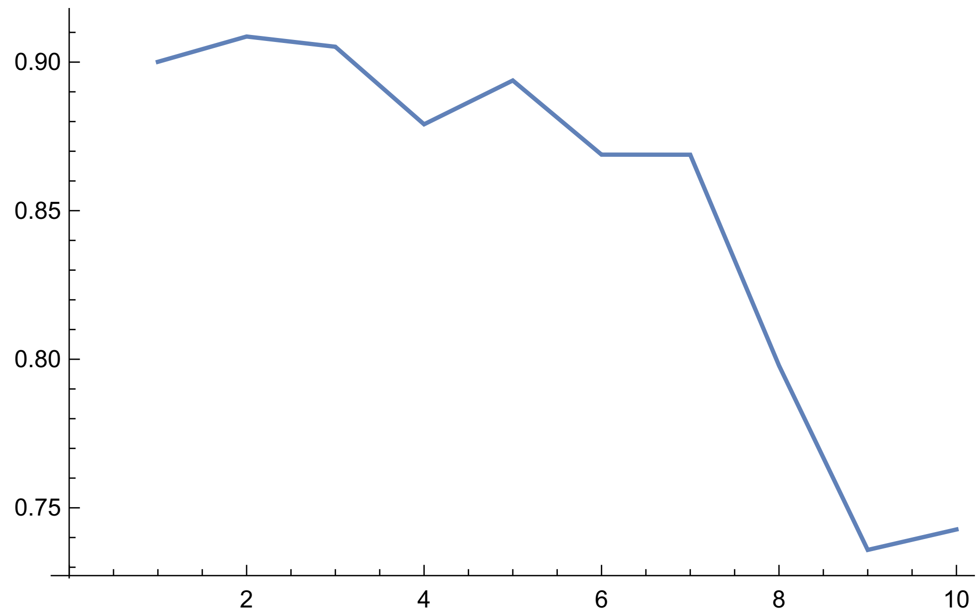


Fig. 2 Service level of two suppliers having to satisfy demand at the warehouse under conditions represented in Fig. 1

Using the closed-form solution provided in (3)-(4), we can also calculate

with $\alpha = 5, \ \beta = 60$

the average service level over the planning horizon $[0,T]$ using (32) in which, at time t, the service level is the ratio between the total supplied quantity and the demand expressed by the warehouse.

$$\text{average service level} = \frac{1}{T}\int_0^T \frac{\sum_{i=1}^{N} x_i(t)}{\xi(t)} dt. \tag{32}$$

The evolution in time of this ratio is provided in Fig. 2. When the CO2 emissions level over the trajectories between the suppliers and the warehouse increases, the service level drops.

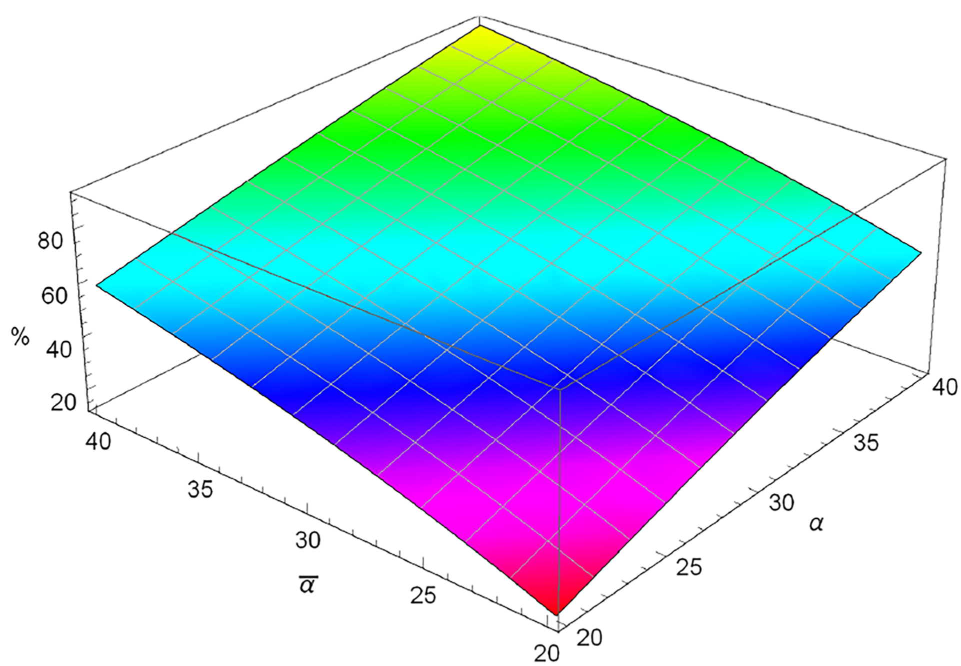


Fig. 3 Envelope of the service level over the planning horizon, in terms of the penalties α and $\bar{\alpha}$

5.2 Multiple Suppliers, a Distribution Center, and Multiple Retail Stores

In the case of a distribution center but various suppliers and stores, the calculation is similar using the result from Theorem 3. In this instance, we used a network of 40 suppliers, 40 stores and a planning horizon [0,20].

In Fig. 3, we represent the evolution of the service level (in percentage) over the considered planning horizon. As expected, the penalties α or $\bar{\alpha}$ increase, the service level increases. Here, we have $\beta = \bar{\beta} = 1$. Note that the maximum service level plateaus at 85%: pollution considerations make filling demand every period not economical. This changes if the penalties increase or pollution trade-off parameter is lowered.

6 Managerial Insights and Conclusion

The present work contributes to the research stream in supply chain management and sustainability literature with the purpose of helping managers in their objective of reducing the carbon footprint of their transport schedules while maximizing service level. In the operations management stream of literature, it answers the call expressed in [24] for further research into extended cooperation between supply chain experts and control engineers to introduce dynamic planning and models to improve performance in logistics systems. It also provides a multi-echelon supply chain model to study carbon footprint for which there is a paucity [14].

As seen in Introduction and already mentioned in [10], supply chain managers must incorporate pollution concerns in their transportation schedules under service level constraints. This becomes especially important as customers' intolerance to delayed delivery [45] and shareholders' concern about carbon emissions increase.

The three models presented cover a variety of network configurations where a manager needs to schedule transport to minimize pollution and yet accommodate the supply and demand over a given planning horizon. Two extensions of the models contemplate lead times and inventory at retail stores. In each configuration, thanks to the linearity of the proposed models, the effect of exogenous shocks can be controlled by means of the expected value as demonstrated by Remark 2.

Further, the advantage of our models is that transport with various carbon footprints can be taken into consideration as well as the possibility that such footprints may evolve in time. As the penalties are exogenous to the models, the supply chain planner can further tweak them under various scenarios to understand the impact of missed, delayed, or anticipated pickups and deliveries on emissions. The solution provided takes into account the planned air pollution in each period and in each location to be traversed. If more than one warehouse has to be taken into account (most networks do have several distribution centers), as many models as there are warehouses can be elaborated since warehouses serve specific disjoined sets of suppliers and delivery end-points.

The use of optimal control theory enables the models to accept any continuous evolution of demand, contract quantity and air pollution emissions. Such flexibility in the use of inputs and variables in transport problems has never before been presented in scientific research centered on transport with the possible exception of Tapiero's work in the 1970s [33–35]. Contrary to various other linear or quadratic programming methods used in other instances of transport optimization, the solution provided using optimal control theory is the optimal one and not an approximation and achieved in a single stage calculation (without iterations).

We purport that our work will be a considerable help to managers or at least to editors of logistic software. We anticipate that the use of the models will include the sensitivity analyses in scenario of varying penalties for missing pickups or deliveries, of pollution impact varying in time and place, of varying suppliers or customers' volumes.

As mentioned in Introduction, stakeholders require that supply chain managers measure the carbon footprint of their logistic operations and reduce it. Strategies which still consider efficiency and cost are out if externalities are not considered. Nowadays, managers have access to more and better data about all of pollution levels, regulations in various regions, as well as transport mode emissions used generate. At times, cities or entire regions are cloaked in particle-filled air due windless periods. Transport in such zones may be curtailed or limited to certain vehicle types (such as electric vehicles). We also expect authorities to tax or otherwise regulate transport emissions through cap-and-trade schemes. In both cases, our methods allow the manager to reschedule the transport plan correspondingly. This will allow for much better overall control of the footprint if the methods presented here are put to use. We expect such big supply networks as big retailers, logistic service providers, and automotive assembly firms to be interested.

There are various limitations to our work. The foremost one is that the models do not account for milk runs when collecting or delivering cargo: transport is considered to consist of single vehicle pick-up and delivery with single origin and destination. Another limitation is the ability of managers to understand the models and their use. The development of a new body of prescriptive knowledge and normative models does not warrant its use by practitioners [46]. In particular, it must be judged according to its pragmatic validity and its practical relevance [47]. Its adoption requires the redesign of an organization and its translation in operators' roles and routines [48, 49]. We expect that examples of such implementations of our proposed methods in real-life settings will also be presented and discussed.

Appendix

Proof of Theorem 1

Proof This is the proof of Theorem 1. The Hamiltonian associated with the problem is:

$$\begin{aligned}
&H(x_1(t), x_2(t), \dots, x_N(t), P(t), \lambda(t)) \\
&= \sum_{j=1}^{N} c_j(t)x_j(t) + \frac{1}{2}\sum_{j=1}^{N} \alpha_j(t)\big(x_j(t) - s_j(t)\big)^2 \\
&\quad + \frac{1}{2}\beta(t)\left(\sum_{j=1}^{N} x_j(t) - \xi(t)\right)^2 dt \\
&\quad + \lambda(t)\left[\sum_{j=1}^{N} \gamma_j(t)x_j(t) - \delta_P P(t)\right]
\end{aligned} \tag{33}$$

The optimality conditions read as:

$$\begin{cases}
\dfrac{\partial H}{\partial x_i} = c_i(t) + \alpha_i(t)\big(x_i(t) - s_i(t)\big) \\
\qquad + \beta(t)\left(\displaystyle\sum_{j=1}^{N} x_j(t) - \xi(t)\right) + \lambda(t)\gamma_i = 0, \quad i = 1 \dots N, \\
\dot{\lambda}(t) = -\dfrac{\partial H}{\partial P} = \delta_P \lambda(t), \\
\dot{P}(t) = \displaystyle\sum_{i=1}^{N} \gamma_i(t)x_i(t) - \delta_P P(t), \\
P(0) = P_0 \geq 0, \\
\lambda(T) = \theta.
\end{cases} \tag{34}$$

The differential equation for λ with terminal condition $\lambda(T) = \theta$, namely

$$\dot{\lambda}(t) = \delta_P \lambda(t), \tag{35}$$

is linear and can be easily solved, leading to $\lambda(t) = \theta e^{\delta_P(t-T)}$. If we plug the expression of λ into the maximum principle, we get:

$$\alpha_i(t)x_i(t) + \beta(t)\sum_{j=1}^{N} x_j(t) = -c_i(t) - \theta\gamma_i e^{\delta_P(t-T)} + s_i(t)\alpha_i(t) + \xi(t)\beta(t). \tag{36}$$

If we define the matrix

$$\Omega(t) := \begin{pmatrix} \alpha_1(t) + \beta(t) & \beta(t) & \dots & \beta(t) \\ \beta(t) & \alpha_2(t) + \beta(t) & \dots & \beta(t) \\ \dots & \dots & \dots & \dots \\ \beta(t) & \beta(t) & \dots & \alpha_N(t) + \beta(t) \end{pmatrix}$$

and the vectors

$$X(t) := \begin{pmatrix} x_1(t) \\ x_2(t) \\ \dots \\ X_N(t) \end{pmatrix}, \quad C(t) := \begin{pmatrix} c_1(t) \\ c_2(t) \\ \dots \\ c_N(t) \end{pmatrix},$$

$$S(t) := \begin{pmatrix} s_1(t)\alpha_1(t) + \xi(t)\beta(t) \\ s_2(t)\alpha_2(t) + \xi(t)\beta(t) \\ \dots \\ s_N(t)\alpha_N(t) + \xi(t)\beta(t) \end{pmatrix}, \text{ and } \Gamma(t) := \begin{pmatrix} \gamma_1(t) \\ \gamma_2(t) \\ \dots \\ \gamma_N(t) \end{pmatrix}.$$

This equation can be written in vectorial form as:

$$\Omega(t)X(t) = -C(t) - \theta e^{\delta_P(t-T)}\Gamma(t) + S(t) \tag{37}$$

and, using the fact that the matrix $\mathbf{\Omega(t)}$ is invertible, we get

$$X(t) = -\Omega(t)^{-1}C(t) - \theta e^{\delta_P(t-T)}\Omega(t)^{-1}\Gamma(t) + \Omega(t)^{-1}S(t). \tag{38}$$

The equation of P boils down to:

$$\dot{P}(t) = \Gamma(t)^T X(t) - \delta_P P(t) \tag{39}$$

and this differential equation has a closed form given by:

$$P(t) = e^{-\delta_P t}\left[\int_0^t \Gamma(s)^T X(s) e^{\delta_P s} ds + P_0\right]. \tag{40}$$

■

Proof of Theorem 2

The proof of this theorem is very similar to the one of Theorem 1 and is therefore omitted.

Proof of Theorem 3

The Hamiltonian associated with the problem is:

$$
\begin{aligned}
&H(x_1(t), x_2(t), \dots, x_N(t), y_1(t), y_2(t), \dots, y_M(t), P(t), \lambda(t)) \\
&= \sum_{i=1}^{N} c_i(t)x_i(t) + \sum_{j=1}^{M} \bar{c}_j(t)y_j(t) + \frac{1}{2}\sum_{i=1}^{N} \alpha_i(t)\big(x_i(t) - s_i(t)\big)^2 \\
&\quad + \frac{1}{2}\sum_{j=1}^{M} \bar{\alpha}_j(t)\big(y_j(t) - \xi_j(t)\big)^2 + \frac{1}{2}\beta(t)\left(\sum_{i=1}^{N} x_i(t) - \xi(t)\right)^2 \\
&\quad + \frac{1}{2}\bar{\beta}(t)\left(\sum_{j=1}^{M} y_j(t) - \sum_{i=1}^{N} x_i(t)\right)^2 \\
&\quad + \lambda(t)\left[\sum_{i=1}^{N} \gamma_i(t)x_i(t) + \sum_{j=1}^{M} \bar{\gamma}_j(t)y_j(t) - \delta_P P(t)\right].
\end{aligned} \tag{41}
$$

The optimality conditions read as:

$$
\begin{cases}
\dfrac{\partial H}{\partial x_i} = c_i(t) + \alpha_i(t)\big(x_i(t) - s_i(t)\big) + \beta(t)\left(\displaystyle\sum_{i=1}^{N} x_j(t) - \xi(t)\right) \\
\qquad - \bar{\beta}(t)\left(\displaystyle\sum_{j=1}^{M} y_j(t) - \sum_{i=1}^{N} x_i(t)\right) + \lambda(t)\gamma_i(t) = 0, \quad i = 1 \dots N \\
\dfrac{\partial H}{\partial y_j} = \bar{c}_j(t) + \bar{\alpha}_j(t)\big(y_j(t) - \xi_j(t)\big) + \bar{\beta}(t)\left(\displaystyle\sum_{j=1}^{M} y_j(t) - \sum_{i=1}^{N} x_i(t)\right) \\
\qquad + \lambda(t)\bar{\gamma}_j(t) = 0, \quad i = j \dots M \\
\dot{\lambda}(t) = -\dfrac{\partial H}{\partial P} = \delta_P \lambda(t), \\
\dot{P}(t) = \displaystyle\sum_{i=1}^{N} \gamma_i(t)x_i(t) + \sum_{j=1}^{M} \bar{\gamma}_j(t)y_j(t) - \delta_P P(t), \\
P(0) = P_0 \geq 0, \\
\lambda(T) = \theta.
\end{cases} \tag{42}
$$

The differential equation for λ with terminal condition $\lambda(T) = \theta$, namely

$$
\dot{\lambda}(t) = \delta_P \lambda(t), \quad \lambda(T) = \theta, \tag{43}
$$

is linear and can be easily solved, leading to $\lambda(t) = \theta e^{\delta_P (t-T)}$. If we plug the expression of λ into the maximum principle, we get:

$$
\begin{aligned}
\alpha_i(t)x_i(t) + (\beta(t) + \bar{\beta}(t))\sum_{i=1}^{N} x_i(t) - \bar{\beta}(t)\sum_{j=1}^{M} y_j(t) = -c_i(t) \\
- \theta\gamma_i(t)e^{\delta_P(t-T)} + s_i(t)\alpha_i(t) + \xi(t)\beta(t), \quad i = 1 \dots N
\end{aligned} \tag{44}
$$

$$
\begin{aligned}
-\bar{\beta}(t)\sum_{i=1}^{N} x_i(t) + \bar{\alpha}_j(t)y_j(t) + \bar{\beta}(t)\sum_{j=1}^{M} y_j(t) = -\bar{c}_j(t) \\
- \theta\bar{\gamma}_j(t)e^{\delta_P(t-T)} + \xi_j(t)\bar{\alpha}_j(t), \quad j = 1 \dots M.
\end{aligned} \tag{45}
$$

If we define the matrix

$$
\Omega 1(t) := \begin{pmatrix} A(t) & B(t) \\ C(t) & D(t) \end{pmatrix} \tag{46}
$$

with

$$
A(t) := \begin{pmatrix}
\alpha_1(t) + \beta(t) + \bar{\beta}(t) & \beta(t) + \bar{\beta}(t) & \dots & \beta(t) + \bar{\beta}(t) \\
\beta(t) + \bar{\beta}(t) & \alpha_2(t) + \beta(t) + \bar{\beta}(t) & \dots & \beta(t) + \bar{\beta}(t) \\
\dots & \dots & \dots & \dots \\
\beta(t) + \bar{\beta}(t) & \beta(t) + \bar{\beta}(t) & \dots & \alpha_N(t) + \beta(t) + \bar{\beta}(t)
\end{pmatrix},
$$

$$
B(t) := \begin{pmatrix}
-\bar{\beta}(t) & \dots & -\bar{\beta}(t) \\
-\bar{\beta}(t) & \dots & -\bar{\beta}(t) \\
\dots & \dots & \dots \\
-\bar{\beta}(t) & \dots & -\bar{\beta}(t)
\end{pmatrix},
$$

$$
C(t) := \begin{pmatrix}
-\bar{\beta}(t) & -\bar{\beta}(t) & \dots & -\bar{\beta}(t) \\
\dots & \dots & \dots & \dots \\
-\bar{\beta}(t) & -\bar{\beta}(t) & \dots & -\bar{\beta}(t)
\end{pmatrix},
$$

$$
D(t) := \begin{pmatrix}
\bar{\alpha}_1(t) + \bar{\beta}(t) & \dots & \bar{\beta}(t) \\
\bar{\beta}(t) & \dots & \bar{\beta}(t) \\
\bar{\beta}(t) & \dots & \bar{\alpha}_M(t) + \bar{\beta}(t)
\end{pmatrix},
$$

and the vectors

$$
U(t) := \begin{pmatrix} x_1(t) \\ x_2(t) \\ \dots \\ x_N(t) \\ y_1(t) \\ y_2(t) \\ \dots \\ y_M(t) \end{pmatrix}, \qquad
C1(t) := \begin{pmatrix} c_1(t) \\ c_2(t) \\ \dots \\ c_N(t) \\ \bar{c}_1(t) \\ \bar{c}_2(t) \\ \dots \\ \bar{c}_M(t) \end{pmatrix},
$$

$$
S1(t) := \begin{pmatrix}
s_1(t)\alpha_1(t) + \xi(t)\beta(t) \\
s_2(t)\alpha_2(t) + \xi(t)\beta(t) \\
\dots \\
s_N(t)\alpha_N(t) + \xi(t)\beta(t) \\
\bar{\alpha}_1(t)\xi_1(t) \\
\bar{\alpha}_2(t)\xi_2(t) \\
\dots \\
\bar{\alpha}_M(t)\xi_M(t)
\end{pmatrix}, \text{ and } \Gamma 1(t) := \begin{pmatrix} \gamma_1(t) \\ \gamma_2(t) \\ \dots \\ \gamma_N(t) \\ \bar{\gamma}_1(t) \\ \bar{\gamma}_2(t) \\ \dots \\ \bar{\gamma}_M(t) \end{pmatrix}.
$$

This equation can be written in vectorial form as:

$$
\Omega 1(t)U(t) = -C1(t) - \theta e^{\delta_P(t-T)}\Gamma 1(t) + S1(t) \tag{47}
$$

and, using the fact that the matrix $\mathbf{\Omega 1(t)}$ is invertible, we get

$$
U(t) = -\Omega 1(t)^{-1}C1(t) - \theta e^{\delta_P(t-T)}\Omega 1(t)^{-1}\Gamma 1(t) + \Omega 1(t)^{-1}S1(t). \tag{48}
$$

The equation of P boils down to:

$$\dot{P}(t) = \Gamma 1(t)^T U(t) - \delta_P P(t) \tag{49}$$

and this differential equation has a closed form given by:

$$P(t) = e^{-\delta_P t}\left[\int_0^t \Gamma 1(s)^T U(s) e^{\delta_P s} ds + P_0\right]. \tag{50}$$

Proof of Theorem 4

The proof of this theorem is very similar to the one of Theorem 1, and it can be obtained from it by introducing a time shift of L units.

Proof of Theorem 5

The proof of this theorem is very similar to the one of Theorem 3, and it can be obtained from this one by adding an exogenous inventory variable.

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Declarations

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References

1. Robbins, N., Tickell, S., & Irwin, W. (2020). Financing climate action with positive social impact: How banking can support a just transition in the UK. resreport, Grantham Research Institute (London School of Economics).
2. Economist, The. (2020). The great disrupter. *The Economist, 436,* 5–10.
3. EPA. (2020). Fast facts on transportation greenhouse gas emissions report#epa-420-f-20-037. *Office of Transportation and Air Quality.* https://tinyurl.com/y453j3jj
4. European Environment Agency. (2019). Co2 emissions from cars: facts and figures. (Ref.: 20190313STO31218) https://tinyurl.com/y293ao8t
5. Büyükkaramikli, N. C., Gürler, Ü., & Alp, O. (2014). Coordinated logistics: joint replenishment with capacitated transportation for a supply chain. *Production and Operations Management, 23*(1), 110–126.
6. Gürbüz, M. Ç., Moinzadeh, K., & Zhou, Y.-P. (2007). Coordinated replenishment strategies in inventory/distribution systems. *Management Science, 53*(2), 293–307.
7. Tempelmeier, H., & Bantel, O. (2015). Integrated optimization of safety stock and transportation capacity. *European Journal of Operational Research, 247*(1), 101–112.
8. Jabali, O., Van Woensel, T., & De Kok, A. (2012). Analysis of travel times and CO2 emissions in time-dependent vehicle routing. *Production and Operations Management, 21*(6), 1060–1074.
9. Van Woensel, T., Creten, R., & Vandaele, N. (2001). Managing the environmental externalities of traffic logistics: The issue of emissions. *Production and Operations Management, 10*(2), 207–223.
10. Franceschetti, A., Honhon, D., Woensel, T. V., Bektaş, T., & Laporte, G. (2013). The time-dependent pollution-routing problem. *Transportation Research Part B: Methodological, 56,* 265–293.
11. Elhedhli, S., & Merrick, R. (2012). Green supply chain network design to reduce carbon emissions. *Transportation Research Part D: Transport and Environment, 17*(5), 370–379.
12. Ma, Q., Wang, W., Peng, Y., & Song, X. (2018). An optimization approach to the intermodal transportation network in fruit cold chain, considering cost, quality degradation and carbon dioxide footprint. *Polish Maritime Research, 25*(1), 61–69.
13. Zhang, D., Zhan, Q., Chen, Y., & Li, S. (2018). Joint optimization of logistics infrastructure investments and subsidies in a regional logistics network with CO2 emission reduction targets. *Transportation Research Part D: Transport and Environment, 60*, 174–190.
14. Xu, Z., Elomri, A., Pokharel, S., & Mutlu, F. (2019). The design of green supply chains under carbon policies: A literature review of quantitative models. *Sustainability, 11*(11), 3094.
15. Presman, E., & Sethi, S. P. (2006). Inventory models with continuous and Poisson demands and discounted and average costs. *Production and Operations Management, 15*(2), 279–293.
16. Levin, Y., McGill, J., & Nediak, M. (2010). Optimal dynamic pricing of perishable items by a monopolist facing strategic consumers. *Production and Operations Management, 19*(1), 40–60.
17. Ciancimino, A., Inzerillo, G., Lucidi, S., & Palagi, L. (1999). A mathematical programming approach for the solution of the railway yield management problem. *Transportation Science, 33*(2), 168–181.
18. Dai, J., Kleywegt, A. J., & Xiao, Y. (2018). Network revenue management with cancellations and no-shows. *Production and Operations Management, 28*(2), 292–318.
19. Kirshner, S. N., & Nediak, M. (2015). Scalable dynamic bid prices for network revenue management in continuous time. *Production and Operations Management, 24*(10), 1621–1635.
20. Li, Y., Jia, G., Cheng, Y., & Hu, Y. (2016). Additive manufacturing technology in spare parts supply chain: a comparative study. *International Journal of Production Research, 55*(5), 1498–1515.
21. Warburton, R. D., Disney, S. M., Towill, D. R., & Hodgson, J. P. (2004). Further insights into 'the stability of supply chains.' *International Journal of Production Research, 42*(3), 639–648.
22. Song, D.-P., & Earl, C. F. (2008). Optimal empty vehicle repositioning and fleet-sizing for two-depot service systems. *European Journal of Operational Research, 185*(2), 760–777.
23. Yang, J., Jaillet, P., & Mahmassani, H. S. (1999). On-line algorithms for truck fleet assignment and scheduling under real-time information. *Transportation Research Record, 1667*(1), 107–113.
24. Ivanov, D., Sethi, S., Dolgui, A., & Sokolov, B. (2018). A survey on control theory applications to operational systems, supply chain management, and industry 4.0. *Annual Reviews in Control, 46*, 134–147.
25. Dolgui, A., Ivanov, D., Sethi, S. P., & Sokolov, B. (2018). Scheduling in production, supply chain and industry 4.0 systems by

optimal control: fundamentals, state-of-the-art and applications. *International Journal of Production Research, 57*(2), 411–432.
26. Zayas-Cabán, G., Lodree, E. J., & Kaufman, D. L. (2020). Optimal control of parallel queues for managing volunteer convergence. *Production and Operations Management*.
27. Anderson, E. G., Jr., Morrice, D. J., & Lundeen, G. (2006). Stochastic optimal control for staffing and backlog policies in a two-stage customized service supply chain. *Production and Operations Management, 15*(2), 262–278.
28. Ivanov, D., Sokolov, B., Chen, W., Dolgui, A., Werner, F., & Potryasaev, S. (2020). A control approach to scheduling flexibly configurable jobs with dynamic structural-logical constraints. *IISE Transactions*, 1–18.
29. Li, Q., Guo, P., Li, C.-L., & Song, J.-S. (2016). Equilibrium joining strategies and optimal control of a make-to-stock queue. *Production and Operations Management, 25*(9), 1513–1527.
30. Papier, F. (2016). Managing electricity peak loads in make-to-stock manufacturing lines. *Production and Operations Management, 25*(10), 1709–1726.
31. Xu, H., Yao, D. D., & Zheng, S. (2011). Optimal control of replenishment and substitution in an inventory system with nonstationary batch demand. *Production and Operations Management, 20*(5), 727–736.
32. Yuan, Q., Chen, Y., Yang, J., & Zhou, Y. (2018). Joint control of emissions permit trading and production involving fixed and variable transaction costs. *Production and Operations Management, 27*(8), 1420–1454.
33. Tapiero, C. (1972). Transportation over time: A systems theory approach. In *Proceedings, TIMS INTL, Israel, 1972*.
34. Tapiero, C., & Soliman, M. (1972). Multi-commodities transportation schedules over time. *Networks, 2*(4), 311–327.
35. Tapiero, C. S. (1971). Transportation-location-allocation problems over time. *Journal of Regional Science, 11*(8), 377–384.
36. Adida, E., & Perakis, G. (2010). Dynamic pricing and inventory control: robust vs. stochastic uncertainty models–a computational study. *Annals of Operations Research, 181*(1), 125–157.
37. Colapinto, C., Jayaraman, R., Ben Abdelaziz, F., & La Torre, D. (2020). Environmental sustainability and multifaceted development: multi-criteria decision models with applications. *Annals of Operations Research, 293*, 405–432.
38. La Torre, D., Liuzzi, D., & Marsiglio, S. (2018). Population and geography do matter for sustainable development. *Environment and Development Economics, 24*(2), 201–223.
39. La Torre, D., Liuzzi, D., & Marsiglio, S. (2021). Transboundary pollution externalities: Think globally, act locally? *Journal of Mathematical Economics, 96,* 102511.
40. La Torre, D., Liuzzi, D., & Marsiglio, S. (2015). Pollution diffusion and abatement activities across space and over time. *Mathematical Social Sciences, 78,* 48–63.
41. La Torre, D., Liuzzi, D., & Marsiglio, S. (2019). The optimal population size under pollution and migration externalities: a spatial control approach. *Mathematical Modelling of Natural Phenomena, 14,* 104.
42. La Torre, D., Liuzzi, D., & Marsiglio, S. (2017). Pollution control under uncertainty and sustainability concern. *Environmental and Resource Economics, 67*(1), 885–903.
43. La Torre, D., Marsiglio, S., & Privileggi, F. (2018). Fractal attractors in economic growth models with random pollution externalities. *Chaos: An Interdisciplinary Journal of Nonlinear Science, 28*(5), 055916.
44. McKinnon, A., & Piecyk, M. (2011) Measuring and managing CO2 emission of European chemical transport. techreport, CEFIC European Chemical Industry Council.
45. Choi, T.-M., Guo, S., Liu, N., & Shi, X. (2020). Optimal pricing in on-demand-service-platform-operations with hired agents and risk-sensitive customers in the blockchain era. *European Journal of Operational Research*.
46. van Aken, J. E. (2005). Management research as a design science: Articulating the research products of mode 2 knowledge production in management. *British Journal of Management, 16*(1), 19–36.
47. Oliva, R. (2019). Intervention as a research strategy. *Journal of Operations Management, 65*(7), 710–724.
48. Reich, J., Kinra, A., Kotzab, H., & Brusset, X. (2020). Strategic global supply chain network design–how decision analysis combining MILP and AHP on a Pareto front can improve decision-making. *International Journal of Production Research, 0*(0), 1–16.
49. van Aken, J. E. (2007). Design science and organization development interventions: Aligning business and humanistic values. *The Journal of Applied Behavioral Science, 43*(1), 67–88.

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